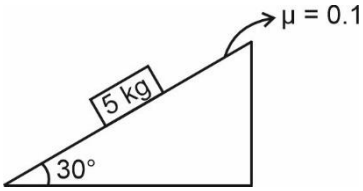
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. For the block shown, F_1 is the minimum force required to move block upwards and F_2 is the minimum force required to prevent it from slipping, find $\left|\vec{F}_1 - \vec{F}_2\right|$



(1) $50\sqrt{3}$ N

(2) $5\sqrt{3}$ N

(3) $25\sqrt{3}$ N

(4) $\frac{5\sqrt{3}}{2}$ N

Answer (2)

Sol. $f_K = \mu mg\cos\theta$

$= 0.1 \times \frac{50\times\sqrt{3}}{2}$

$= 2.5\sqrt{3}$ N

$F_1 = mg\sin\theta + f_K$

$= 25 + 2.5\sqrt{3}$

$F_2 = mg\sin\theta - f_K$

$= 25 - 2.5\sqrt{3}$

$\therefore \quad F_1 - F_2 = 5\sqrt{3}$ N

2. Force on a particle moving in straight line is given by $\vec{F} = 6t^2\hat{i} - 3t\hat{j}$ and velocity is $\vec{v} = 3t^2\hat{i} + 6t\hat{j}$. Find power at $t = 2$.

(1) 216 W

(2) 108 W

(3) 0 W

(4) 54 W

Answer (1)

Sol. $P = \vec{F}\cdot\vec{v}$

$= 18t^4 - 18t^2$

$\Rightarrow \quad P(t=2) = 18[16-4] = 216$ W

3. If $E = \frac{A - x^2}{Bt}$ where E is energy, x is displacement and t is time. Find dimensions of AB

(1) $[M^{-1}L^2T]$

(2) $[ML^2T^{-1}]$

(3) $[M^{-1}L^2T^{-2}]$

(4) $[ML^2T^{-2}]$

Answer (1)

Sol. $[A] = L^2$

$B = \frac{x^2}{tE} \equiv \frac{L^2}{TML^2T^{-2}} = \frac{1}{MT^{-1}}$

$[B] = M^{-1}T$

$[AB] = [M^{-1}L^2T]$

4. Unpolarised light incident on transparent glass at incident angle 60°. If reflected ray is completely polarised, then angle of refraction is

(1) 45°

(2) 60°

(3) 30°

(4) 37°

Answer (3)

Sol. By Brewsters law

$\mu = \tan i$

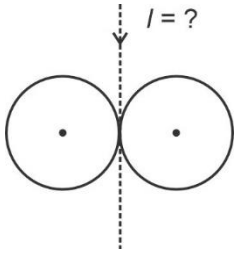
$\mu = \sqrt{3}$

$\therefore \quad 1\times\frac{\sqrt{3}}{2} = \sqrt{3}\times\sin r$

$\sin r = \frac{1}{2}$

$r = 30°$

5. Two solid spheres each of mass 2 kg and radius 75 cm are arranged as shown. Find MOI of the system about the given axis.



(1) 3.15 kg m^2

(2) 31.5 kg m^2

(3) 0.9 kg m^2

(4) 9 kg m^2

Answer (1)

Sol. $I = \left(\frac{2}{5}MR^2 + MR^2\right) \times 2$

$= \frac{14}{5} \times 2 \times \frac{9}{16}$

$= \frac{63}{20}$

= 3.15 kg m^2

6. If the current through an incandescent lamp decreases by 20%, how much change will be there in its illumination?

(1) 36% (2) 64%

(3) 20% (4) 40%

Answer (1)

Sol. $p = i^2R$

$p' = 0.64\, i^2R$

7. Find the speed of sound in oxygen gas at STP.

(1) 300 m/s

(2) 350 m/s

(3) 330 m/s

(4) 400 m/s

Answer (3)

Sol. $v = \sqrt{\frac{\gamma RT}{M}} = 330$ m/s

8. Find average power in electric circuit if source voltage (V) = 20sin(100ωt) and current in the circuit $(I) = 2\sin(100\omega t + \frac{\pi}{3})$

(1) 10 W (2) 20 W

(3) 5 W (4) 15.5 W

Answer (1)

Sol. $\langle P \rangle = IV\cos\phi$

$= \frac{20}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \cos 60°$

= 10 W

9. In a photoelectric experiment, frequency $f = 1.5f_0$ (f_0: threshold frequency). If the frequency of light is changed to $f/2$, then photocurrent becomes (intensity of light has doubled)

(1) Zero

(2) Doubled

(3) Same

(4) Thrice

Answer (1)

Sol. Since $\frac{f}{2} < f_0$

$\Rightarrow$ current = 0

10. Radius of curvature of equiconvex lens is 20 cm. Material of lens is having refractive index of 1.5. Find image distance from lens if an object is placed 10 cm away from the lens.

(1) 20 cm

(2) 10 cm

(3) 40 cm

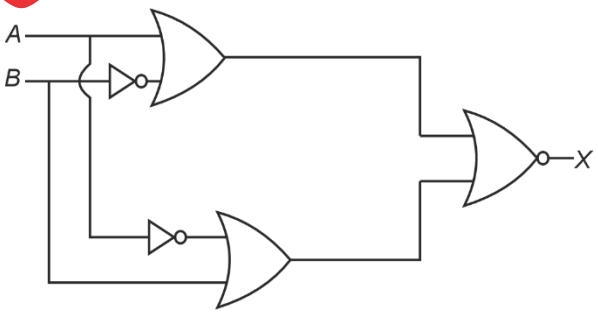
(4) 5 cm

Answer (1)

Sol. $\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$ $f = 20$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} + \frac{1}{10} = \frac{1}{20}$

11. 

Draw truth table of given gate circuit.

(1)

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

(2)

A	B	X
0	0	0
0	1	0
1	0	0
1	1	0

(3)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(4)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

Answer (2)

Sol. $X = \overline{(A+\bar{B})+(\bar{A}+B)}$

$\overline{(A+\bar{B})}\cdot\overline{(\bar{A}+B)}$

$(\bar{A}\cdot\bar{\bar{B}})\cdot(\bar{\bar{A}}\cdot\bar{B})$

$(\bar{A}\cdot B)\cdot(A\cdot\bar{B}) = \bar{A}\cdot B\cdot A\cdot\bar{B} = 0$

12. The magnetic flux through a loop varies with time as $\phi = 5t^2 - 3t + 5$. If the resistance of loop is 8 Ω, find the current through it at t = 2 s

(1) $\frac{15}{8}$ A

(2) $\frac{5}{8}$ A

(3) $\frac{17}{8}$ A

(4) $\frac{13}{8}$ A

Answer (3)

Sol. $\frac{d\phi}{dt} = 10t - 3$

at t = 2, V = 17

$i = \frac{V}{R} = \frac{17}{8}$ A

13. 8 moles of oxygen and 4 moles of nitrogen are at same temperature T and are mixed. The total internal energy is

(1) 60RT

(2) 15RT

(3) 30RT

(4) 90RT

Answer (3)

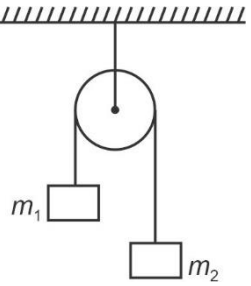
Sol. $U = nC_vT$

$\Rightarrow U = n_1C_{v_1}T + n_2C_{v_2}T$

$\Rightarrow 8\times\frac{5R}{2}\times T + 4\times\frac{5R}{2}\times T$

$= 30RT$

14. In the system shown below, the pulley 4 string are ideal. If the acceleration of blocks is $\frac{g}{8}$, find $\frac{m_1}{m_2}$



(1) $\frac{9}{7}$

(2) $\frac{8}{7}$

(3) $\frac{5}{7}$

(4) $\frac{9}{8}$

Answer (1)

Sol. $a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{g}{8}$

$8m_1 - 8m_2 = m_1 + m_2$

$7m_1 = 9m_2$

$\frac{m_1}{m_2} = \frac{9}{7}$

15. The force between two charged particle placed in air at separation x is F_0. Both the charged particle immerged in a medium of dielectric constant K without changing separation between two charge, then net force on one of the particle is now

(1) $\frac{F_0}{K}$

(2) $\frac{F_0}{2K}$

(3) $\frac{2F_0}{K}$

(4) F_0

Answer (1)

Sol. In air $F = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r_2}$

In medium $F' = \frac{1}{4\pi(k \in_0)}\frac{q_1 q_2}{r^2}$

$F' = \frac{F_0}{K}$

16. Two vector each of magnitude A are inclined at angle θ with each other, then magnitude of resultant vector is

(1) $A\cos^2\frac{\theta}{2}$

(2) $2A\cos\frac{\theta}{2}$

(3) $2A\cos\theta$

(4) $A\cos\frac{\theta}{2}$

Answer (2)

Sol. The magnitude of resultant vector (R) $= \sqrt{a^2 + b^2 + 2ab\cos\theta}$

here $a = b = A$

then $R = \sqrt{A^2 + A^2 + 2A^2\cos\theta}$

$= A\sqrt{2}\sqrt{1+\cos\theta}$

$= \sqrt{2}A\sqrt{2\cos^2\frac{\theta}{2}}$

$= 2A\cos\frac{\theta}{2}$

17. **Statement 1 :** Electric and magnetic energy density in electromagnetic waves are equal.

Statement 2 : Electromagnetic waves exert pressure on a surface.

(1) Statement 1 is true & Statement 2 is true and is correct explanation of Statement 1

(2) Statement 1 is true & Statement 2 is true but is not correct explanation of Statement 1

(3) Statement 1 is true but Statement 2 is false

(4) Statement 1 is false but Statement 2 is true

Answer (2)

Sol. $\frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$

$\because E = CB$ and $C = \frac{1}{\mu_0\varepsilon_0}$

18. A pendulum completes 50 oscillations in 40 seconds. If the length of pendulum is (20 $\pm$ 0.2) cm and resolution of watch is 1 second, find the percentage error in calculation of g.

(1) 7%

(2) 3%

(3) 6%

(4) 4%

Answer (3)

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

$g = \frac{4\pi^2 l}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$

$= \frac{0.2}{20} + 2\left(\frac{1}{40}\right)$

$= 6\%$

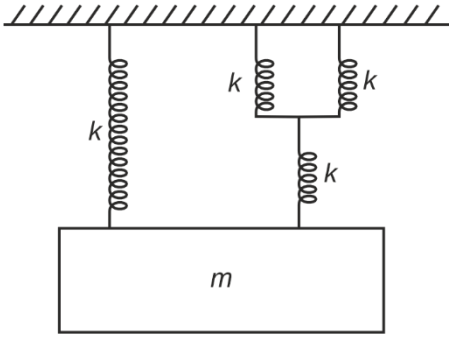
19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The period of oscillation of system shown below is $\pi\sqrt{\frac{\alpha m}{5k}}$ then α is _____



Answer (12)

Sol. $k_{eq} = \frac{2k \cdot k}{3k} + k = \frac{5k}{3}$

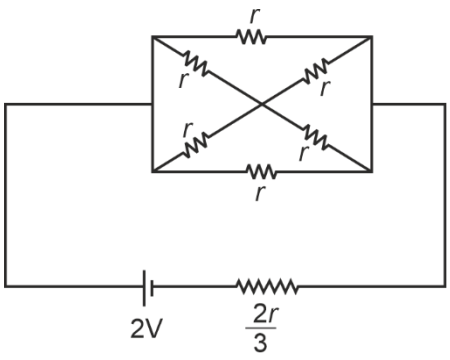
Angular frequency of oscillation (ω) = $\sqrt{\frac{k_{eq}}{m}}$

$\omega = \sqrt{\frac{5k}{3m}}$

Period of oscillation (τ) = $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3m}{5k}}$

$= \pi\sqrt{\frac{12m}{5k}}$

22. In the given circuit, $r = 2\ \Omega$. The power dissipated in the circuit is _____W.



Answer (2)

Sol. $R_{eq} = r$

$\therefore \quad P = \frac{V^2}{r} = \frac{4}{2} = 2\ \text{W}$

23. A body of mass m is projected with speed u at angle 45° with horizontal. The angular momentum of body, about point of projection when body is at highest point, is $\frac{\sqrt{2}\, m\, u^3}{xg}$ find x,

Answer (8)

Sol. $L = mu\cos\theta\frac{u^2\sin^2\theta}{2g}$

$= mu^3\frac{1}{4\sqrt{2}\,g} \Rightarrow x = 8$

24. Mass of moon is $\frac{1}{81}$ times the mass of a planet and radius is $\frac{1}{9}$ times the radius of the planet. The ratio of escape speed from planet to escape speed from moon is _____.

Answer (3)

Sol. $v_{esc} = \sqrt{\frac{2GM}{R}}$

$\Rightarrow$ Ratio $= \sqrt{\frac{81}{9}} = 3$

25. Find the mass number of an atom whose radius is half of that of a given atom of mass number 192.

Answer (24)

Sol. $r = R_0(192)^{\frac{1}{3}}$

$\frac{r}{2} = R_0(m)^{\frac{1}{3}}$

$m = \frac{192}{8} = 24$

26.

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. **Statement 1 :** S_8 disproportionate into $H_2S_2O_3$ and S^{2-} in alkaline medium

   Statement 2 : ClO_4^- undergoes disproportionation in acidic medium.

   (1) Statement 1 is correct but statement 2 is incorrect

   (2) Statement 1 is incorrect but statement 2 is correct

   (3) Both statement 1 and statement 2 are correct

   (4) Both statement 1 and statement 2 are incorrect

Answer (1)

Sol. (1) $\overset{0}{S_8} + NaOH \rightarrow \overset{-2}{Na_2S} + \overset{+2}{Na_2S_2O_3}$ (Oxidation / Reduction)

(2) Cl is in its highest oxidation state (+7). It cannot be further oxidised

Therefore, statement 1 is correct but statement 2 is incorrect.

2. Which of the following is correct?

   (1) $[NiCl_4]^{2-}$ – diamagnetic
   $[Ni(CO)_4]$ – diamagnetic

   (2) $[Ni(CO)_4]$ – diamagnetic
   $[NiCl_4]^{2-}$ – paramagnetic

   (3) $[NiCl_4]^{2-}$ – paramagnetic
   $[Ni(CO)_4]$ – paramagnetic

   (4) $[NiCl_4]^{2-}$ – paramagnetic
   $[Ni(CO)_4]$ – diamagnetic

Answer (2)

Sol. Ni^{2+} : $4s^0 3d^8$ (No pairing with Cl^-)

$[Ni(CO)_4]$: $4s^0 3d^{10}$ (diamagnetic)

3. **Statement-I :** Among 15th group hydrides reducing character decreases from NH_3 to BiH_3.

   Statement-II : E_2O_3 and E_2O_5 are always basic.

   [Where E is group 15 element]

   (1) Both statement-I and Statement-II are correct

   (2) Statement-I is correct and Statement-II is false

   (3) Statement-I is false and Statement-II is correct

   (4) Both Statement-I and Statement-II are false

Answer (4)

Sol. Reducing character increases from NH_3 to BiH_3.

Group 15 oxides of type E_2O_3 and E_2O_5 are not always basic.

4. Which of the following has maximum ionic character?

   (1) KCl (2) AgCl

   (3) $CoCl_2$ (4) $BaCl_2$

Answer (1)

Sol. Polarisation power $\propto \frac{\text{Charge}}{\text{Size}}$

for K^+, polarising power is least and ionic character is maximum.

5. Match the following :

   (a) $[Cr(H_2O)_6]^{+3}$ (i) $t_{2g}^2 eg^0$

   (b) $[Fe(H_2O)_6]^{+3}$ (ii) $t_{2g}^3 eg^0$

   (c) $[Ni(H_2O)_6]^{+2}$ (iii) $t_{2g}^3 eg^2$

   (d) $[V(H_2O)_6]^{+3}$ (iv) $t_{2g}^6 eg^2$

   (1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

   (2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

   (3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

   (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Answer (1)

Sol. (a) $[Cr(H_2O)_6]^{+3} \rightarrow Cr^{+3} \rightarrow t_{2g}^3 eg^0$

(b) $[Fe(H_2O)_6]^{+3} \rightarrow Fe^{3+} \rightarrow t_{2g}^3 eg^2$

(c) $[Ni(H_2O)_6]^{+2} \rightarrow Ni^{2+} \rightarrow t_{2g}^6 eg^2$

(d) $[V(H_2O)_6]^{+3} \rightarrow V^{3+} \rightarrow t_{2g}^2 eg^0$

6. Quantum number for outermost electron of K-atom are given by

(1) $n = 4, l = 0, m = 0, s = \frac{1}{2}$

(2) $n = 4, l = 1, m = 0, s = \frac{1}{2}$

(3) $n = 3, l = 0, m = 0, s = \frac{1}{2}$

(4) $n = 4, l = 0, m = 1, s = \frac{1}{2}$

Answer (1)

Sol. $K_{19} = 1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$

For 4*s* electron

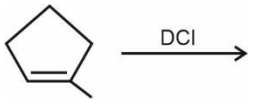
$n = 4$

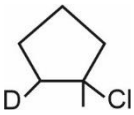
$l = 0$

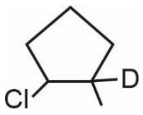
$m = 0$

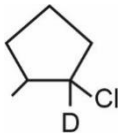
$s = \frac{1}{2}$

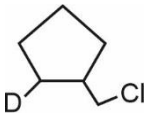
7. What is the product formed in the below given reaction?



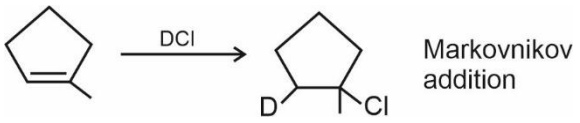
(1) 

(2) 

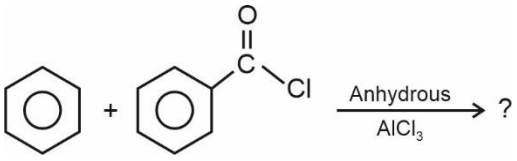
(3) 

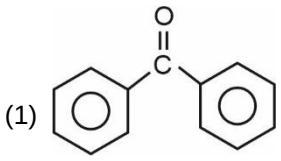
(4) 

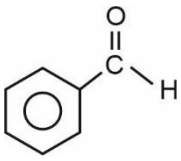
Answer (1)

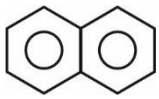
Sol. 

8. What is the major product formed in the following reaction?



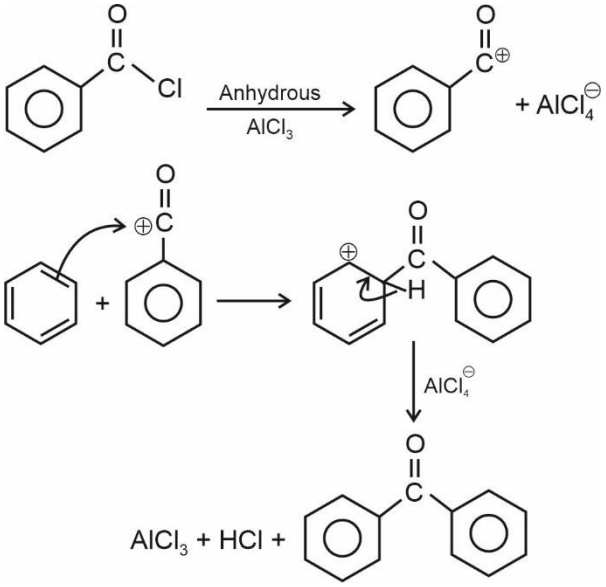
(1) 

(2) 

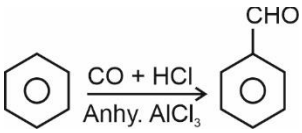
(3) 

(4) 

Answer (1)

Sol. 

9. Identify the given rection



(1) Rosenmund reaction

(2) Stephen reaction

(3) Gattemann Koch reaction

(4) Etard reaction

Answer (3)

Sol. The given reaction is Gattemann Koch reaction.

10. Choose the correct answers.

(A) Mn_2O_7 is a oil at room temperature.

(B) V_2O_4 react with acid to give VO^{2+}

(C) CrO is a basic oxide

(D) V_2O_5 does not react with acids.

(1) A, B and C only (2) B, C and D only

(3) A only (4) B and C only

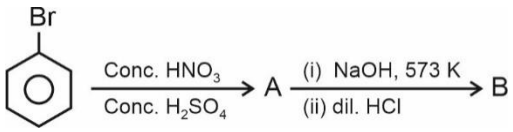
Answer (1)

Sol. A, B and C are correct.

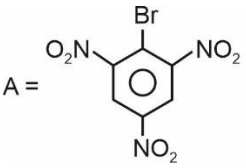
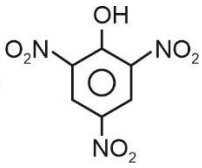
- Mn_2O_7 is a green oil at room temperature.
- V_2O_4 react with acids to give VO^{2+}.
- CrO is Basic and CrO_3 is acidic.
- V_2O_5 react with acids as well as alkali.

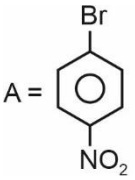
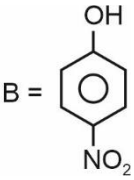
(Ref. NCERT Pg 224)

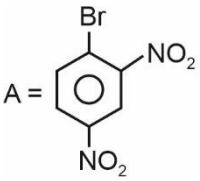
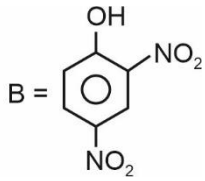
11. Consider the following reaction :

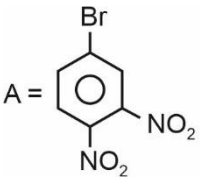
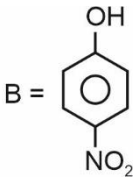


A and B respectively are

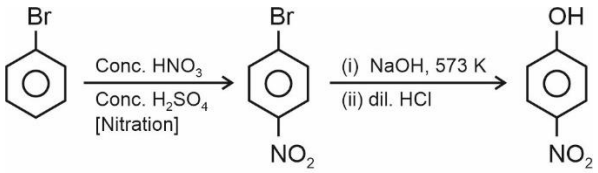
(1) A =  , B = 

(2) A =  , B = 

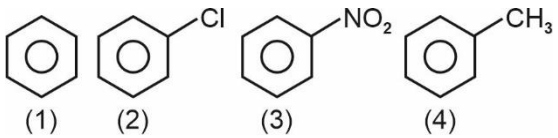
(3) A =  , B = 

(4) A =  , B = 

Answer (2)

Sol. 

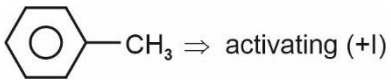
12. What will be the reactivity order of following compounds towards electrophilic substitution reaction?



(1) 1 > 3 > 2 > 4 (2) 4 > 1 > 2 > 3

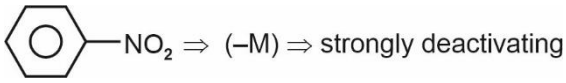
(3) 3 > 2 > 1 > 4 (4) 4 > 3 > 1 > 2

Answer (2)

Sol.  CH_3 ⇒ activating (+I)

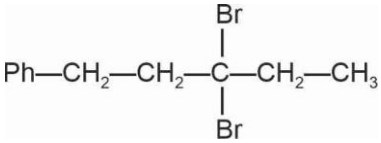
 H ⇒ Neutral (No effect)

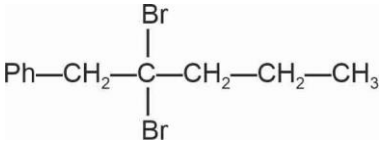
 Cl ⇒ Weakly deactivating (–I dominates over +M)

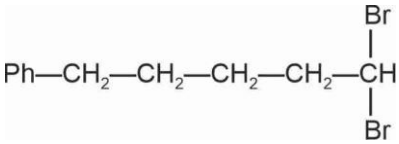
 NO_2 ⇒ (–M) ⇒ strongly deactivating

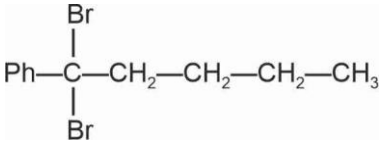
13. Correct IUPAC structure for the given organic compound is

2,2-Dibromo-1-phenylpentane

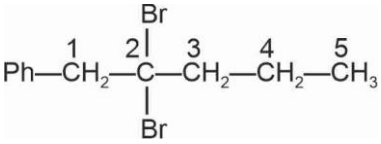
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

14. **Statement-I :** Aniline on reaction with concentrated H_2SO_4 at 475 K gives p-amino benzene sulphonic acid. This gives blood red colour with Lassaigne's test.

Statement-II : Aniline forms a salt with anhydrus $AlCl_3$ in Friedel Craft's reaction.

(1) Both Statement-I and Statement-II are correct

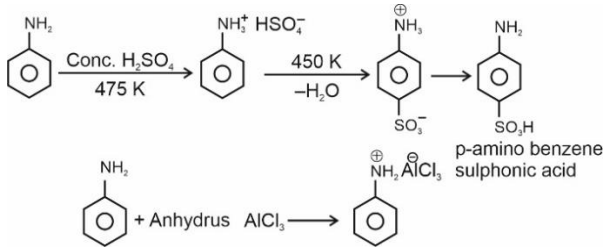
(2) Both Statement-I and Statement-II are incorrect

(3) Statement-I is correct and Statement-II incorrect

(4) Statement-I is incorrect and Statement-II correct

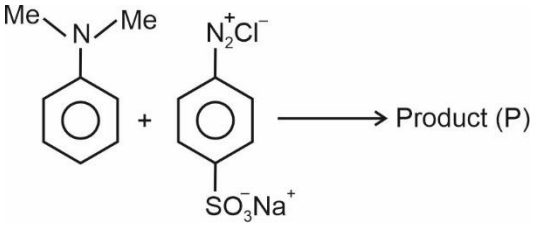
Answer (1)

Sol.



p-amino benzene sulphonic acid contains both N and S, so it gives blood red colour with Lassaigne's test.

15. Consider the following reaction.

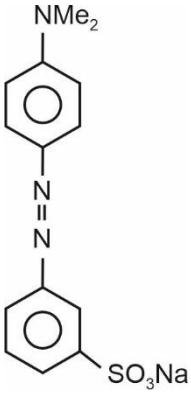


Select P

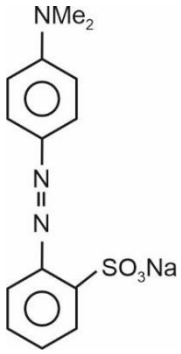
(Where Me is CH_3)

(1)

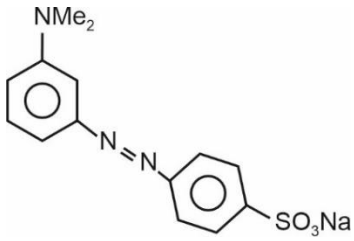
(2)



(3)

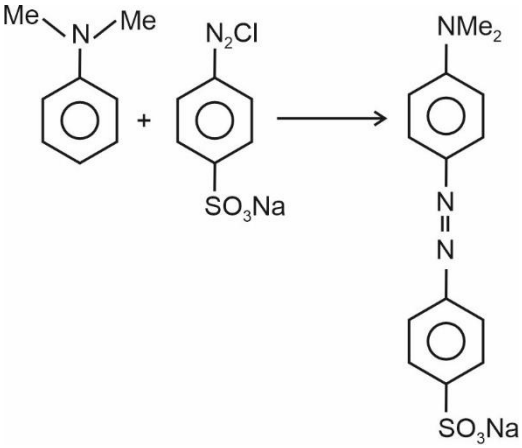


(4)



Answer (1)

Sol.



is an example of azo coupling reaction and final product is methyl orange.

16. $A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$

In the about reaction, the correct relation between K_p, α and equilibrium pressure (p) is

(1) $K_p = \frac{\alpha^{1/2}\, 2p^{1/2}}{(2+\alpha)^{1/2}}$

(2) $K_p = \frac{\alpha^{1/2}\, p^{3/2}}{(2+\alpha)^{3/2}}$

(3) $K_p = \frac{\alpha^{1/2}\, 2p^{1/2}}{(2+\alpha)^{3/2}}$

(4) $K_p = \frac{\alpha^{3/2}\, p^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$

Answer (4)

Sol.

$$A(g) \rightleftharpoons B(g) + \frac{1}{2}(g)$$

Initial moles	n	0	0
Eqb. moles	$n(1-\alpha)$	$n\alpha$	$\frac{n\alpha}{2}$

total moles $= n\left(1+\frac{\alpha}{2}\right)$

Eqb. pressure	$\frac{(1-\alpha)p}{1+\frac{\alpha}{2}}$	$\frac{\alpha p}{1+\frac{\alpha}{2}}$	$\frac{\left(\frac{\alpha}{2}\right)p}{1+\frac{\alpha}{2}}$

$$K_p = \frac{\frac{\alpha p}{\left(1+\frac{\alpha}{2}\right)} \times \left[\frac{\alpha p}{(2+\alpha)}\right]^{\frac{1}{2}}}{\frac{(1-\alpha)p}{1+\frac{\alpha}{2}}}$$

$$K_p = \frac{\alpha^{3/2}\, p^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$$

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Half life of a first order reaction is 36 hr. Find out time (in hr) required for concentration of reactant to get reduced by 90%.

Answer (120)

Sol. $t_{90} = \frac{2.303}{k}\log\left(\frac{100}{100-90}\right)$

$$= \frac{2.303\times 36}{2.303\times \log 2}\times \log 10 = \frac{36}{0.3} = 120$$

22. A 1 mol ideal gas expands from 10 L to 100 L at 300 k, if above expansion takes place reversibly and isothermally then magnitude of work done is _____ (in KJ)

Answer (06)

Sol. $w = -nRT\ln\frac{V_2}{V_1}$

$$|w| = 2.303\, nRT\log\frac{V_2}{V_1}$$

$$|w| = 1\times 2.303\times 8.314\times 300\log\frac{100}{10}$$

$|w| = 5744$ J

$|w| = 5.744$ kJ ≈ 6 kJ

23. How many of the following vitamins are stored in Human Body?

A, B, C, D, E, K?

Answer (4)

Sol. A, D, E, K vitamins are fat soluble vitamins, are stored in liver and adipose tissue.

While vitamin B and vitamin C are water soluble and must be supplied regularly in diet (not stored) (except vitamin B_{12}) (NCERT, Pg : 426)

24. Number of moles of H^+ required by 1 mole MnO_4^- to oxidize oxalate ion to CO_2 is____.

Answer (8)

Sol. The balanced reaction is as follows

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

2 mole MnO_4^- react with 16 mole H^+

1 mole MnO_4^- will react with 8 mole H^+

25. The potassium chloride is heated with potassium dichromate and conc. sulphuric acid to give products. The oxidation state of chromium in product is (+)_____.

Answer (06.00)

Sol. This is an example of chromyl chloride test

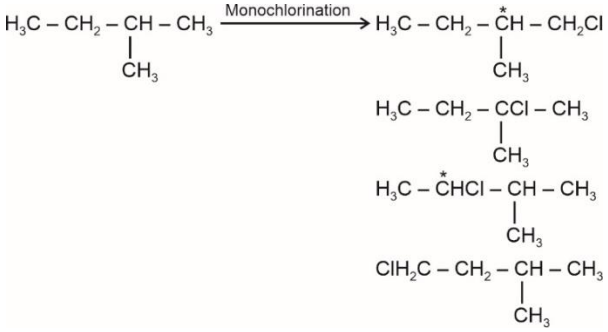
$$K_2Cr_2O_7 + 4KCl + 6H_2SO_4 \rightarrow 6KHSO_4 + 2CrO_2Cl_2 + 3H_2O$$

Oxidation state of Cr is +6.

26. Number of structural isomeric products formed by monochlorination of 2-methylbutane in presence of sunlight is_____.

Answer (4)

Sol



27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. $a = \sin^{-1}(\sin 5), b = \cos^{-1}(\cos 5)$ then $a^2 + b^2$ is equal to

(1) $8\pi^2 - 40\pi + 50$ (2) $4\pi^2 + 25$

(3) $8\pi^2 - 50$ (4) $8\pi^2 + 40\pi + 50$

Answer (1)

Sol. $a = \sin^{-1}(\sin 5) = 5 - 2\pi$

and $b = \cos^{-1}(\cos 5) = 2\pi - 5$

$\therefore\ a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2$

$= 8\pi^2 - 40\pi + 50$

2. A coin is biased such that head has two chances than tails, what is the probability of getting 2 heads and 1 tail?

(1) $\frac{1}{29}$ (2) $\frac{2}{29}$

(3) $\frac{1}{9}$ (4) $\frac{4}{9}$

Answer (4)

Sol. Let probability of tail is $\frac{1}{3}$

$\Rightarrow$ Probability of getting head = $\frac{2}{3}$

$\therefore$ Probability of getting 2 heads and 1 tail

$= \left(\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3}\right) \times 3$

$= \frac{4}{27} \times 3$

$= \frac{4}{9}$

3. Let mean and variance of 6 observations a, b, 68, 44, 40, 60 be 55 and 194. If $a > b$ then find $a + 3b$

(1) 211.83 (2) 201.59

(3) 189.57 (4) 198.87

Answer (2)

Sol. $\frac{a + b + 68 + 44 + 40 + 60}{6} = 55$

$212 + a + b = 330$

$\Rightarrow\ a + b = 118$

$\frac{\sum x_i^2}{n} - \left(\bar{x}\right)^2 = 194$

$\frac{a^2 + b^2 + (68)^2 + (44)^2 + (40)^2 + (60)^2}{6} - (55)^2 = 194$

$= 3219$

$11760 + a^2 + b^2 = 19314$

$\Rightarrow\ a^2 + b^2 = 19314 - 11760$

$= 7554$

$(a + b)^2 - 2ab = 7554$

From here $b = 41.795$

$a + b = 118$

$\Rightarrow\ a + b + 2b = 118 + 83.59$

$= 201.59$

4. If 2nd, 8th, 44th terms of A.P. are 1st, 2nd and 3rd terms respectively of G.P. and first term of A.P. is 1 then the sum of first 20 terms of A.P. is

(1) 970 (2) 916

(3) 980 (4) 990

Answer (1)

Sol. $a + d$, $a + 7d$ and $a + 43d$ are 1st, 2nd, 3rd term of G.P.

$\frac{a + 7d}{a + d} = \frac{a + 43d}{a + 7d}$

$\Rightarrow\ (a + 7d)^2 = (a + d)(a + 43d)$

$\Rightarrow\ a^2 + 49d^2 + 14d = a^2 + 44ad + 43d^3$

$\Rightarrow\ 6d^2 = 30ad$

$\Rightarrow\ d^2 = 5d$

$\Rightarrow\ d = 0, 5$

$a = 1, d = 5$

$S_{20} = \frac{20}{2}[2 + (19)5]$

$= 10\ [95 + 2]$

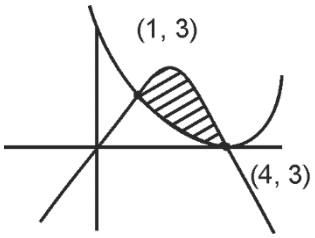
$= 970$

5. The area of the region enclosed by the parabolas $y = 4 - x^2$ and $3y = (x-4)^2$ is in (sq. unit)?

(1) $\frac{14}{3}$ (2) 4

(3) $\frac{32}{3}$ (4) 6

Answer (4)



Sol. Area $= \left|\int_1^4 \left[(4-x)^2 - \frac{(x-4)^2}{3}\right] dx\right|$

Area $= \left|4x - \frac{x^3}{3} - \frac{(x-4)^3}{9}\right|_1^4$

$= \left|\left(16 - \frac{64}{3}\right) - \left(4 - \frac{1}{3} + \frac{27}{9}\right)\right|$

$= \left|16 - \frac{64}{3} - 4 + \frac{1}{3} + 3\right|$

$= |15 - 2| = 6$

6. If $A\begin{bmatrix}1\\0\\1\end{bmatrix} = 2\begin{bmatrix}1\\0\\1\end{bmatrix}$, $A\begin{bmatrix}-1\\0\\1\end{bmatrix} = 4\begin{bmatrix}-1\\0\\1\end{bmatrix}$

and $A\begin{bmatrix}0\\1\\0\end{bmatrix} = 2\begin{bmatrix}0\\1\\0\end{bmatrix}$ where, A is a 3×3 matrix and

$(A - 3I)\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$ then the value of (x, y, z) is

(1) (1, 2, 3) (2) (1, –2, 3)

(3) (1, –2, –3) (4) (–1, –2, –3)

Answer (3)

Sol. Let $A = \begin{bmatrix}x_1 & y_1 & z_1\\x_2 & y_2 & z_2\\x_3 & y_3 & z_3\end{bmatrix}$

Given $A\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}2\\0\\2\end{bmatrix}$ ... (1)

$\therefore \begin{bmatrix}x_1 + z_1\\x_2 + z_2\\x_3 + z_3\end{bmatrix} = \begin{bmatrix}2\\0\\2\end{bmatrix}$

$\therefore x_1 + z_1 = 2$ ... (2)

$x_2 + z_2 = 0$ ... (3)

$x_3 + z_3 = 0$ ... (4)

Given $A = \begin{bmatrix}-1\\0\\1\end{bmatrix} = \begin{bmatrix}-4\\0\\4\end{bmatrix}$

$\therefore \begin{bmatrix}-x_1 + z_1\\-x_2 + z_2\\-x_3 + z_3\end{bmatrix} = \begin{bmatrix}4\\0\\4\end{bmatrix}$

$\Rightarrow -x_1 + z_1 = -4$ ... (5)

$-x_2 + z_2 = 0$ ... (6)

$-x_3 + z_3 = 4$ ... (7)

Given $A = \begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\2\\0\end{bmatrix}$

$\therefore \begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix} = \begin{bmatrix}0\\2\\0\end{bmatrix}$

$\therefore y_1 = 0, y_2 = 2, y_3 = 0$

$\therefore$ from (2), (3), (4), (5), (6) and (7)

$x_1 = 3, x_2 = 0, x_3 = -1$

$y_1 = 0, y_2 = 2, y_3 = 0$

$z_1 = -1, z_2 = 0, z_3 = 3$

$\therefore A = \begin{bmatrix}3 & 0 & -1\\0 & 2 & 0\\-1 & 0 & 3\end{bmatrix}$

$\therefore$ Now $(A - 3I)\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$

$\therefore \begin{bmatrix}0 & 0 & -1\\0 & -1 & 0\\-1 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$

$\begin{bmatrix}-z\\-y\\-x\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$

$[z = 1], [y = -2], [x = -3]$

7. Let $f : R \to (0, \infty)$ be increasing function such that $\lim\limits_{x\to\infty} \frac{f(7x)}{f(x)} = 1$ then $\lim\limits_{x\to\infty}\left(\frac{f(5x)}{f(x)} - 1\right)$ is equal to

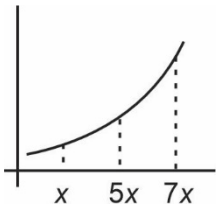
(1) Zero (2) 4

(3) 1 (4) $\frac{4}{5}$

Answer (1)

Sol. f is increasing function

$x < 5x < 7x$



$f(x) < f(5x) < f(7x)$

$\frac{f(x)}{f(x)} < \frac{f(5x)}{f(x)} < \frac{f(7x)}{f(x)}$

$\lim\limits_{x\to\infty}\frac{f(x)}{f(x)} < \lim\limits_{x\to\infty}\frac{f(5x)}{f(x)} < \lim\limits_{x\to\infty}\frac{f(7x)}{f(x)}$

$1 < \lim\limits_{x\to\infty}\frac{f(5x)}{f(x)} < 1 \Rightarrow \lim\limits_{x\to\infty}\frac{f(5x)}{f(x)} = 1$

$\lim\limits_{x\to\infty}\left(\frac{f(5x)}{f(x)} - 1\right) = 0$

8. Let z_1 and z_2 be two complex numbers such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$, then the value of $\left|z_1^4 + z_2^4\right|$ is equal to

(1) 75 (2) $25\sqrt{5}$

(3) $15\sqrt{15}$ (4) $30\sqrt{3}$

Answer (1)

Sol. $z_1 + z_2 = 5$

$z_1^3 + z_2^3 = 20 + 15i$

$z_1^3 + z_2^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$

$z_1^3 + z_2^3 = 125 - 3z_1 \cdot z_2(5)$

$\Rightarrow$ $20 + 15i = 125 - 15z_1z_2$

$\Rightarrow$ $3z_1z_2 = 25 - 4 - 3i$

$3z_1z_2 = 21 - 3i$

$z_1 \cdot z_2 = 7 - i$

$(z_1 + z_2)^2 = 25$

$z_1^2 + z_2^2 = 25 - 2(7 - i)$

$= 11 + 2i$

$\left(z_1^2 + z_2^2\right)^2 = 121 - 4 + 44i$

$\Rightarrow$ $z_1^4 + z_2^4 + 2(7 - i)^2 = 117 + 44i$

$\Rightarrow$ $z_1^4 + z_2^4 = 117 + 44i - 2(49 - 1 - 14i)$

$= 21 + 72i$

$\Rightarrow$ $\left|z_1^4 + z_2^4\right| = 75$

9. The number of solutions of equation $e^{\sin x} - 2e^{-\sin x} = 2$ is

(1) More than 2 (2) 2

(3) 1 (4) 0

Answer (4)

Sol. Take $e^{\sin x} = t$ $(t > 0)$

$\Rightarrow t - \frac{2}{t} = 2$

$\Rightarrow \frac{t^2 - 2}{t} = 2$

$\Rightarrow t^2 - 2t - 2 = 0$

$\Rightarrow t^2 - 2t + 1 = 3$

$\Rightarrow (t - 1)^2 = 3$

$\Rightarrow t = 1 \pm \sqrt{3}$

$\Rightarrow t = 1 \pm 1.73$

$\Rightarrow t = 2.73$ or -0.73 (rejected as $t > 0$)

$\Rightarrow e^{\sin x} = 2.73$

$\Rightarrow \log_e e^{\sin x} = \log_e 2.73$

$\Rightarrow \sin x = \log_e 2.73 > 1$

So no solution.

10. The line passes through the centre of circle $x^2 + y^2 - 16x - 4y = 0$, it interacts with the positive coordinate axis at A & B. Then find the minimum value of $OA + OB$, where O is origin.

(1) 20 (2) 18

(3) 12 (4) 24

Answer (1)

Sol. $(y - 2) = m(x - 8)$

$\Rightarrow$ x-intercept

$\Rightarrow$ $\left(\frac{-2}{m} + 8\right)$

$\Rightarrow$ y-intercept

$\Rightarrow$ $(-8m + 2)$

$\Rightarrow$ $OA + OB = \frac{-2}{m^2} + 8 - 8m + 2$

$f'(m) = \frac{2}{m^2} - 8 = 0$

$\Rightarrow$ $m^2 = \frac{1}{4}$

$\Rightarrow \quad m = \frac{-1}{2}$

$\Rightarrow \quad f\left(\frac{-1}{2}\right) = 18$

$\Rightarrow$ Minimum = 18

11. If for some m, n; ${}^6C_m + 2\left({}^6C_{m+1}\right) + {}^6C_{m+2} > {}^8C_3$ and ${}^{n-1}P_3 : {}^nP_4 = 1:8$, then ${}^nP_{m+1} + {}^{n+1}C_m$ is equal to

(1) 6756 (2) 7250

(3) 6223 (4) 6550

Answer (1)

Sol. ${}^6C_m + 2\left({}^6C_{m+1}\right) + {}^6C_{m+2} > {}^8C_3$

${}^7C_{m+1} + {}^7C_{m+2} > {}^8C_3$

${}^8C_{m+2} > {}^8C_3$

$\therefore \quad m = 2$

and ${}^{n-1}P_3 : {}^nP_4 = 1:8$

$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{8}$

$\therefore \quad n = 8$

$\therefore \quad {}^nP_{m+1} + {}^{n+1}C_m = {}^8P_5 + {}^9C_2$

$= 8\times7\times6\times5\times4 + \frac{9\times8}{2}$

= 6756

12. Let $f:(-\infty,-1] \to (a, b]$ be defined as $f(x) = e^{x^3-3x+1}$, if f is both one and onto, then the distance from a point $P(2a+4, b+2)$ to curve $x + ye^{-3} - 4 = 0$ is

(1) $\sqrt{e^3+2}$ (2) $\frac{e^3+2}{\sqrt{e^3+1}}$

(3) $\frac{e^3+2}{\sqrt{e^6+1}}$ (4) e

Answer (3)

Sol. $f(x) = e^{x^3-3x+1}$

$f'(x) = e^{x^3-3x+1}\cdot(3x^2-3)$

$= e^{x^2-3x+1}\cdot3(x-1)(x+1)$

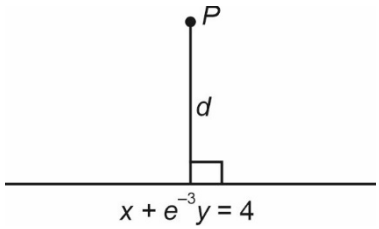
For $x \in (-\infty,-1], f'(x) \ge 0$

$\therefore$ $f(x)$ is increasing function

$\therefore a = e^{-\infty} = 0 = f(-\infty)$

$b = e^{-1+3+1} = e^3 = f(-1)$

$\therefore P(4, e^3+2)$



$d = \frac{(e^3+2)(e^{-3})}{\sqrt{1+e^{-6}}} = \frac{1+2e^{-3}}{\sqrt{1+e^{-6}}} = \frac{e^3+2}{\sqrt{e^6+1}}$

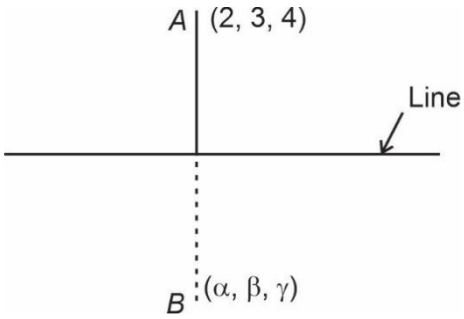
13. If (α, β, γ) is mirror image of the point (2, 3, 4) with respect to the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Then $2\alpha + 3\beta + 4\gamma$ is

(1) 29 (2) 30

(3) 31 (4) 32

Answer (1)

Sol.



Take $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$x = 2\lambda+1, \quad y = 3\lambda+2, \quad z = 4\lambda+3$

$\overrightarrow{AB} = (\alpha-2)\hat{i} + (\beta-3)\hat{j} + (\gamma-4)\hat{k}$

Now,

$(\alpha-2)\cdot2 + (\beta-3)\cdot3 + (\gamma-4)\cdot4 = 0$

$2\alpha - 4 + 3\beta - 9 + 4\gamma - 16 = 0$

$\Rightarrow \quad 2\alpha + 3\beta + 4\gamma = 29$

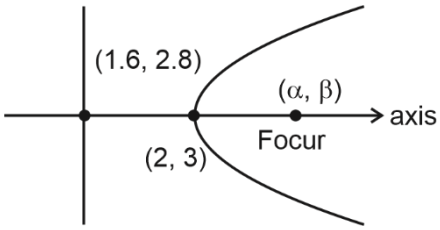
14. A parabola has vertex (2, 3), equation of directrix is $2x - y = 1$ and equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, e = \frac{1}{\sqrt{2}}$ and ellipse passing through focur of parabola then square of length of latus rectum of ellipse is

(1) $\frac{6564}{25}$ (2) $\frac{3288}{25}$

(3) $\frac{6272}{25}$ (4) $\frac{4352}{25}$

Answer (4)

Sol.



Slope of axis $=\frac{1}{2}$

$y-3=\frac{1}{2}(x-2)$

$\Rightarrow \quad 2y-6=x-2$

$\Rightarrow \quad 2y-x-4=0$

$2x+y-6=0$

$4x+2y-12=0$

$\alpha+1.6=4 \Rightarrow \alpha=2.4$

$\beta+2.8=6 \Rightarrow \beta=3.2$

Ellipse passes through (2.4, 3.2)

$$\Rightarrow \quad \frac{\left(\frac{24}{10}\right)^2}{a^2}+\frac{\left(\frac{32}{10}\right)^2}{b^2}=1$$

Also $1-\frac{a^2}{b^2}=\frac{1}{2}$

$$\frac{a^2}{b^2}=\frac{1}{2}$$

$$\frac{144}{25}b^2+\frac{256}{25}a^2=a^2b^2$$

$$\frac{144}{25}+\frac{256}{25}\times\frac{1}{2}=a^2$$

$$\Rightarrow \quad \frac{(128+144)}{25}=a^2 \Rightarrow \frac{272}{25}=a^2$$

$$\Rightarrow \quad b^2=\frac{2\times 272}{25}$$

Latus rectum $=\frac{2b^2}{a}$

(Latus rectum)^2

$$=\frac{4b^4}{a^2}=4\left(\frac{b^2}{a^2}\right)b^2=\frac{8\times 272\times 2}{25}=\frac{4352}{25}$$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The value of $\frac{120}{\pi^3}\left|\int_0^{\pi}\frac{x^2\sin x\cdot\cos x}{(\sin x)^4+(\cos x)^4}dx\right|$ is

Answer (15)

Sol. $\int_0^{\pi}\frac{x^2\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}dx$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\left(x^2-(\pi-x)^2\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin x\cdot\cos x(2\pi x-\pi^2)}{\sin^4 x+\cos^4 x}x$$

$$=2\pi\int_0^{\frac{\pi}{2}}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx-\pi^2\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$=2\pi\cdot\frac{\pi}{4}\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx-\pi^2\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$=-\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$=-\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x dx}{1-2\sin^2 x+\cos^2 x}$$

$$=-\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}}\frac{\frac{1}{2}\sin 2x}{1-\frac{1}{2}\sin^2 2x}dx$$

$$=-\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}}\frac{\sin 2x}{2-\sin^2 2x}dx$$

$$=-\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}}\frac{\sin 2x}{1+\cos^2 2x}dx$$

Let $\cos 2x = t$

$$= -\frac{\pi^2}{2}\int_{1}^{-1} -\frac{1}{2}\frac{dt}{1+t^2}$$

$$= -\frac{\pi^2}{4}\int_{-1}^{1} \frac{dt}{1+t^2}$$

$$= -\frac{\pi^2}{4}\cdot\frac{\pi}{2} = -\frac{\pi^3}{8}$$

$$\therefore \quad \frac{120}{\pi^3}\left|-\frac{\pi^3}{8}\right| = 15$$

22. The number of ways to distribute the 21 identical apples to three children's so that each child gets at least 2 apples.

Answer (136)

Sol. After giving 2 apples to each child 15 apples left now 15 apples can be distributed in ${}^{15+3-1}C_2 = {}^{17}C_2$ ways

$$= \frac{17\times 16}{2} = 136$$

23. If $A = \{1, 2, 3, \ldots 100\}$, $R = \{(x, y) \mid 2x = 3y, x, y \in A\}$ is symmetric relation on A and the number of elements in R is n, the smallest integer value of n is

Answer (0)

Sol. $\because$ R is symmetric relation

$\Rightarrow$ $(y, x) \in R \,\forall\, (x, y) \in R$

$(x, y) \in R \Rightarrow 2x = 3y$ and $(y, x) \in R \Rightarrow 3x = 2y$

Which holds only for (0, 0)

Which does not belongs to R.

$\therefore$ Value of $n = 0$

24. Matrix A of order 3×3 is such that $|A| = 2$ if $n = \underbrace{\left|\text{adj}\left(\text{adj}\left(\text{adj}\ldots(a)\right)\right)\right|}_{2024 \text{ times}}$ then remainder when n is divided by 9 is

Answer (7)

Sol. $|A| = 2$

$$\underbrace{\text{adj}\left(\text{adj}\left(\text{adj}\ldots(a)\right)\right)}_{2024 \text{ times}} = |A|^{(n-1)^{2024}}$$

$$= |A|^{2^{2024}}$$

$$= 2^{2^{2024}}$$

$$2^{2024} = \left(2^2\right)2^{2022} = 4(8)^{674} = 4(9-1)^{674}$$

$$\Rightarrow \quad 2^{2024} \equiv 4 \pmod 9$$

$$\Rightarrow \quad 2^{2024} \equiv 9m + 4, \quad m \leftarrow \text{even}$$

$$2^{9m+4} \equiv 16\cdot\left(2^3\right)^{3m} \equiv 16 \pmod 9$$

$$\equiv 7$$

25.

26.

27.

28.

29.

30.

❑ ❑ ❑